

# A novel conditional diagnosability algorithm under the PMC model<sup>①</sup>

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## Abstract

Conditionally t-diagnosable and t-diagnosable are important in system level diagnosis. Therefore, it is valuable to identify whether the system is conditionally t-diagnosable or t-diagnosable and derive the corresponding conditional diagnosability and diagnosability. In the paper, distinguishable measures of pairs of distinct faulty sets with a new perspective on establishing functions are focused. Applying distinguishable function and decision function, it is determined whether a system is conditionally t-diagnosable (or t-diagnosable) or not under the PMC (Preparata, Metze, and Chien) model directly. Based on the decision function, a novel conditional diagnosability algorithm under the PMC model is introduced which can calculate conditional diagnosability rapidly.

**Key words:** the PMC (Preparata, Metze, and Chien) model, conditionally t-diagnosable, conditional diagnosability, conditional diagnosability algorithm

## 0 Introduction

With the continuous development of large-scale integration, multiprocessor computer systems can consist of hundreds of processors. However, the high complexity of those systems may threaten their reliability. To resolve this issue, in 1967, Preparata, Metze, and Chien presented the definition of system level diagnosis and proposed a so-called PMC model and t-diagnosable<sup>[1]</sup>. In 1992, Sengupta and Dahbura proposed that the most important necessary and sufficient condition for t-diagnosable was that each pair of distinct faulty sets should be distinguishable, provided the number of faulty vertices was no more than  $t$ <sup>[2]</sup>.

Lai, et al.<sup>[3]</sup> introduced the conditional diagnosability based on the assumption that all neighbors of any processor in a multiprocessor system could not be fault simultaneously. A system is conditionally t-diagnosable if each pair of conditional faulty sets is distinguishable. Thus far, distinguishability of a pair of distinct faulty sets is widely adopted in the study of t-diagnosable<sup>[2,4,5]</sup>, conditionally t-diagnosable<sup>[3,5-9]</sup>, strong diagnosability<sup>[5,9]</sup> and g-good-neighbor conditional t-diagnosable<sup>[10]</sup>. However, lacking of distinguishable measures has caused bad influence.

In this paper, distinguishable measures of pairs of distinct faulty sets with a new perspective on establishing functions is focused. After a distinguishable func-

tion and a decision function are constructed, how to identify whether a system is conditionally t-diagnosable (t-diagnosable) or not under the PMC model is studied. Finally, a novel algorithm is given to derive conditional diagnosability under the PMC model.

## 1 Preliminaries

A multiprocessor computer system consisting of  $n$  processors is modeled as a graph where each vertex represents a processor and each edge represents a link. Let  $G(V, E)$  be such a graph. An edge  $(u, v) \in E(G)$ , with  $u, v \in V(G)$ , is a test edge of  $G(V, E)$ , which represents a test performed by  $u$  on  $v$ . The outcome of edge  $(u, v)$ , denoted by  $\sigma(u, v)$ , is "0" if  $u$  evaluates  $v$  as a pass and "1" if  $u$  evaluates  $v$  as a fault. An outcome is reliable only if the tester is fault-free. The collection of all test outcomes in  $G(V, E)$  is called a syndrome, denoted by  $\sigma$ . Each vertex has two states: fault-free and faulty. If vertex  $u$  is identified as fault-free, then denoted by  $u = 0$ ; otherwise  $u = 1$ .

In the PMC model, each vertex  $u$  is able to test another vertex if there is a link between them. The outcome of a test performed by a fault-free tester is 1 (respectively, 0) if the tested vertex is faulty (respectively, fault-free), whereas the outcome of a test performed by a faulty tester is unreliable. Table 1 summarizes the invalidation rules for the PMC model.

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Table 1 Invalidation rules for the PMC model

$u$	$v$	$\sigma(u, v)$
0	0	0
0	1	1
1	0	0 or 1
1	1	0 or 1

Some known results about faulty set and t-diagnosable are listed below.

**Definition 1**<sup>[4]</sup>: A subset  $F \subseteq V(G)$  is called a faulty set of a given syndrome  $\sigma$ , for any  $(u, v) \in E(G)$  and  $u \in V(G) - F$ ,  $\sigma(u, v) = 0$  if  $v \in V(G) - F$ ,  $\sigma(u, v) = 1$  if  $v \in F$ .

For a given syndrome  $\sigma$ , a faulty set  $F \subseteq V(G)$  is said to be consistent with  $\sigma$  if  $F$  can produce  $\sigma$ . Let  $\sigma(F)$  represent the set of syndromes which can be produced if  $F$  is the set of faulty vertices.

**Definition 2**<sup>[1]</sup>: A system is a t-diagnosable one if and only if, for a given syndrome  $\sigma$ , all the faulty vertices can be identified that the number of faulty vertices are not more than  $t$ .

**Definition 3**<sup>[2]</sup>: Two distinct faulty sets  $F_1$  and  $F_2$  are said to be indistinguishable if  $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$ ; otherwise,  $(F_1, F_2)$  is distinguishable.

According to Definition 2 and 3, the following two lemmas about t-diagnosable are proposed.

**Lemma 1**<sup>[4]</sup>: For a pair of distinct faulty sets  $F_1$  and  $F_2$ , with  $F_1 \subseteq V(G)$  and  $F_2 \subseteq V(G)$ ,  $(F_1, F_2)$  is distinguishable if there exists at least one test from  $V(G) - (F_1 \cup F_2)$  to  $F_1 \Delta F_2$ . Operator  $\Delta$  implies exclusive-or (XOR). Hence,  $F_1 \Delta F_2 = (F_1 - F_2) \cup (F_2 - F_1)$ . The operator  $||$  implies cardinality. Then,  $|F_1|$  is the cardinality of  $F_1$ .

**Lemma 2**<sup>[2]</sup>: A system is t-diagnosable if each pair of distinct faulty sets  $F_1$  and  $F_2$  is distinguishable, provided that  $|F_1| \leq t$  and  $|F_2| \leq t$ .

Diagnosability is an important measure of self-diagnostic capability. The diagnosability of system  $G$  is the maximum value of  $t$  such that  $G$  is t-diagnosable, written as  $t(G)$ .

Motivated by the deficiency of classical measurement of diagnosability, Lai, et al. presented conditional diagnosability by claiming the property that each vertex had at least one fault-free neighbor<sup>[3]</sup>. Then, they introduced some useful definitions and lemmas as follows.

**Definition 4**<sup>[3]</sup>: Faulty set  $F \subseteq V(G)$  is a conditional faulty set only if every vertex of the system has at least one fault-free neighbor.

**Lemma 3**<sup>[3]</sup>: A system is conditionally t-diagnosable if each pair of distinct conditional faulty sets  $(F_1, F_2)$  is distinguishable, with  $|F_1| \leq t$  and  $|F_2| \leq t$ .

**Definition 5**<sup>[3]</sup>: The conditional diagnosability of system  $G$  is the maximum value of  $t$  that  $G$  is conditionally t-diagnosable, denoted as  $t_c(G)$ .

In this paper, an undirected diagnosable system is adopted, which assumes that every test edge is bidirectional. The undirected diagnosable system is a special diagnosable system. An arbitrary edge  $(u, v)$  of an undirected diagnosable system implies that  $u$  can test  $v$  and  $v$  can test  $u$  too.

## 2 Distinguishable measure of pairs of distinct faulty sets

As mentioned above, t-diagnosable and conditionally t-diagnosable are closely related to the distinguishability of pairs of distinct faulty sets. Therefore, an interesting question arises here: how to identify whether two distinct faulty sets are distinguishable or not. In this section, some important theorems and lemmas about distinguishable measures of two distinct faulty sets will be presented.

**Theorem 1**: Let  $F_1$  and  $F_2$  be two distinct faulty sets of an undirected diagnosable system,  $(F_1, F_2)$  is distinguishable, then there exists at least one undirected edge  $(u, v)$ , such  $(u + v) |_{F_1} + (u + v) |_{F_2} = 1$ .  $(u + v) |_{F_x}$  is the sum of  $u$  and  $v$  when  $F_x$  is the set of faulty vertices,  $(u + v) |_{F_x} = (u) |_{F_x} + (v) |_{F_x}$ ,  $(u) |_{F_x} = 0$  if  $u \notin F_x$ , and  $(u) |_{F_x} = 1$  if  $u \in F_x$ . According to the definition of  $(u + v) |_{F_x}$ ,  $(u + v) |_{F_1} = 1$  (or  $(u + v) |_{F_2} = 1$ ) implies that  $u + v = 1$ , which means one of  $\{u, v\}$  is fault-free and the other is faulty, when  $F_1$  (or  $F_2$ ) is the current faulty vertices set.

**Proof**: This theorem is proved by contradiction. For each undirected edge  $(u, v)$  of the system, it is assumed  $(u + v) |_{F_1} + (u + v) |_{F_2} \neq 1$ . Without loss of generality, there exists 7 cases. As shown in Table 2, only case 2 lacks the possibility of satisfying  $\sigma(u, v) |_{F_1} = \sigma(u, v) |_{F_2}$  and  $\sigma(v, u) |_{F_1} = \sigma(v, u) |_{F_2}$ , which means  $\sigma(F_1) \cap \sigma(F_2) = \emptyset$ .

According to  $(u + v) |_{F_1} + (u + v) |_{F_2} \neq 1$ , case 2 will not appear in the system. Therefore, the system has the possibility of satisfying  $\sigma(u, v) |_{F_1} = \sigma(u, v) |_{F_2}$  and  $\sigma(v, u) |_{F_1} = \sigma(v, u) |_{F_2}$ , which implies  $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$ . According to Definition 3,  $(F_1, F_2)$  is an indistinguishable pair of faulty sets, which contradicts the assumption. The theorem follows.

It is easy to prove that  $(u + v) |_{F_1} + (u + v) |_{F_2} = 1$  is another form of the existence of at least one test edge from  $V - (F_1 \cup F_2)$  to  $(F_1 \Delta F_2)$ . Therefore, Theorem 1 is also proved by Lemma 1.

Table 2 The value of  $(u + v) \mid_{F_1} + (u + v) \mid_{F_2}$  under all possible scenarios

Faulty set	$(u) \mid_{F_x}$	$(v) \mid_{F_x}$	$\sigma(u, v) \mid_{F_x}$	$\sigma(v, u) \mid_{F_x}$
$x = 0$	0	0	0	0
$x = 1$	0	0	0	0
$(u + v) \mid_{F_1} + (u + v) \mid_{F_2} = 0$				
Case 1				
Faulty set	$(u) \mid_{F_x}$	$(v) \mid_{F_x}$	$\sigma(u, v) \mid_{F_x}$	$\sigma(v, u) \mid_{F_x}$
$x = 0$	0	0	0	0
$x = 1$	1	0	$x$	1
$(u + v) \mid_{F_1} + (u + v) \mid_{F_2} = 1$				
Case 2				
Faulty set	$(u) \mid_{F_x}$	$(v) \mid_{F_x}$	$\sigma(u, v) \mid_{F_x}$	$\sigma(v, u) \mid_{F_x}$
$x = 0$	0	0	0	0
$x = 1$	1	1	$x$	$x$
$(u + v) \mid_{F_1} + (u + v) \mid_{F_2} = 2$				
Case 3				
Faulty set	$(u) \mid_{F_x}$	$(v) \mid_{F_x}$	$\sigma(u, v) \mid_{F_x}$	$\sigma(v, u) \mid_{F_x}$
$x = 0$	1	0	$x$	1
$x = 1$	1	0	$x$	1
$(u + v) \mid_{F_1} + (u + v) \mid_{F_2} = 2$				
Case 4				
Faulty set	$(u) \mid_{F_x}$	$(v) \mid_{F_x}$	$\sigma(u, v) \mid_{F_x}$	$\sigma(v, u) \mid_{F_x}$
$x = 0$	1	0	$x$	1
$x = 1$	0	1	1	$x$
$(u + v) \mid_{F_1} + (u + v) \mid_{F_2} = 2$				
Case 5				
Faulty set	$(u) \mid_{F_x}$	$(v) \mid_{F_x}$	$\sigma(u, v) \mid_{F_x}$	$\sigma(v, u) \mid_{F_x}$
$x = 0$	1	0	$x$	1
$x = 1$	1	1	$x$	$x$
$(u + v) \mid_{F_1} + (u + v) \mid_{F_2} = 3$				
Case 6				
Faulty set	$(u) \mid_{F_x}$	$(v) \mid_{F_x}$	$\sigma(u, v) \mid_{F_x}$	$\sigma(v, u) \mid_{F_x}$
$x = 0$	1	1	$x$	$x$
$x = 1$	1	1	$x$	$x$
$(u + v) \mid_{F_1} + (u + v) \mid_{F_2} = 4$				
Case 7				

According to Theorem 1, an important distinguishable function is presented which can identify whether a pair of faulty sets is distinguishable or not.

**Definition 6:** For any two distinct subsets  $F_1$  and  $F_2$ , distinguishable function  $D(F_1, F_2) =$

$$\prod_{each(u,v) \in E(G)} ((u + v) \mid_{F_1} + (u + v) \mid_{F_2} - 1).$$

According to Definition 6,  $D(F_i, F_j) = D(F_j, F_i)$  is got. To avoid double-counting,  $i < j$  is set.

**Lemma 4:**  $D(F_1, F_2) = 0$  represents that  $(F_1, F_2)$  is distinguishable; otherwise,  $(F_1, F_2)$  is indistinguishable.

**Proof:** Obviously, according to Theorem 1,  $(F_1, F_2)$  is distinguishable which can be expressed as there exists at least one undirected edge  $(u, v)$ , such  $(u + v) \mid_{F_1} + (u + v) \mid_{F_2} = 1$ . Therefore,  $(u + v) \mid_{F_1} + (u + v) \mid_{F_2} - 1 = 0$  can be deduced. That is to say,  $(F_1, F_2)$  is distinguishable and can be expressed as

$$\prod_{each(u,v) \in E} ((u + v) \mid_{F_1} + (u + v) \mid_{F_2} - 1) = 0, (u, v) \in E. \text{ Hence, Lemma 4 holds.}$$

According to Lemma 2 and Lemma 3, t-diagnosable and conditionally t-diagnosable are tied to distinguishability of pairs of distinct faulty sets and conditional faulty sets, respectively.

Next, a decision function will be provided which can decide whether the system is t-diagnosable (or conditionally t-diagnosable).

**Definition 7:** Decision function  $J(F_1, F_2, \dots, F_m) = \sum_{1 \leq i < j \leq m} \mid D(F_i, F_j) \mid$ , where  $F_1, F_2, \dots, F_m$  are all the possible faulty sets (or conditional faulty sets) with  $\mid F_1 \mid, \mid F_2 \mid, \dots, \mid F_m \mid \leq t$ .

**Lemma 5:**  $J(F_1, F_2, \dots, F_m) = 0$  represents the fact that the system is t-diagnosable (or conditionally t-diagnosable), where  $F_1, F_2, \dots, F_m$  are all the possible faulty sets (or conditional faulty sets) with  $\mid F_1 \mid, \mid F_2 \mid, \dots, \mid F_m \mid \leq t$ ; otherwise, the system is not t-diagnosable (or conditionally t-diagnosable).

**Proof:** By Definition 7,  $J(F_1, F_2, \dots, F_m) = 0$  means  $D(F_i, F_j) = 0$  for  $1 \leq i < j \leq m$ . According to Lemma 4,  $D(F_i, F_j) = 0$  represents the fact that  $(F_i, F_j)$  is distinguishable. By Lemma 2, the system is t-diagnosable. Hence, the lemma holds.

The decision function  $J(F_1, F_2, \dots, F_m)$  can be used in both t-diagnosable systems and conditionally t-diagnosable systems. The only difference is whether  $F_1, F_2, \dots, F_m$  are all the possible faulty sets or all the possible conditional faulty sets.

### 3 A novel conditional diagnosability algorithm under the PMC model

The conditional diagnosability algorithm under the proposed PMC model is based on Theorem 1 and decision function  $J(F_1, F_2, \dots, F_m)$ . The effectiveness of this conditional diagnosability algorithm has been confirmed by Lemma 5. Above all, all the possible conditional faulty sets of the system must be derived. Then, the decision function  $J(F_1, F_2, \dots, F_m)$  is called to identify whether the system is conditionally t-diagnosable or not and then obtain conditional diagnosability. The new algorithm can be outlined as follows:

**Step 1:** Construct conditional faulty set equations.

For each vertex  $u \in V$ , we set  $\Gamma(u) = \{u' \in V | (u, u') \in E\}$ . According to the definition of conditional diagnosability,  $\Gamma(u)$  has at least one fault-free neighbor that can be denoted by  $\Gamma(u) = u_1 u_2 \dots u_q = 0$ . The equations of all the vertices in the system compose the conditional faulty set equations.

Step 1 can be described by the following pseudocode.

---

**Input:**  $G(V, E)$   
**Output:** The conditional faulty set equations

---

```

1 for every vertex  $u \in V(G)$ 
2   Compute  $\Gamma(u) = \{u_1, u_2, \dots, u_q\}$ 
3   To build equation  $\prod_{i=1}^q u_i = 0$ 
4 end for
5 Collects all equations to form conditional faulty set equations
6 return to Step 2
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**Step 2:** Convert each equation of the conditional faulty set equations into a relational table.

For example, the equation  $x_1 x_2 \dots x_q = 0$  means that there exists at least one vertex “0”. The relational table corresponding to  $x_1 x_2 \dots x_q = 0$  is Table 3, which consists of  $2^q - 1$  tuples.

Table 3 The relational table corresponds to  $x_1 x_2 \dots x_q = 0$

$x_1$	$x_2$	...	$x_q$
0	0	...	0
1	0	...	0
...	...	...	...
1	1	...	0

Step 2 can be described as follows:

---

**Input:** Conditional fault model equations  
**Output:** Relational tables  $X_1, X_2, \dots, X_m$

---

```

1 for every equation of equations
2   Transform equation into a relation table  $X_i$ 
3    $i = i + 1$ 
4 end for
5 return to Step 3
```

---

**Step 3:** Derive all the possible conditional faulty sets.

After all the conditional faulty set equations have been converted into relational tables, all the possible conditional faulty sets in this step will be derived. Let all of the relational tables be  $X_1, X_2, \dots, X_r$ .

First of all, empty relational table  $X$  is defined. If relational tables  $X$  and  $X_1$  have one or more fields in

common, then the two tables are joined as a new relational table  $X$  by natural join ( $\bowtie$ ), denoted by  $X = X \bowtie X_1$ , otherwise, they are joined by Cartesian product ( $\times$ ), denoted by  $X = X \times X_1$ . Repeat this step from  $X_2$  to  $X_r$ . The final new relational table  $X$  is the set of all the possible conditional faulty sets, denoted by  $X = \{F_1, F_2, \dots, F_m\}$ .

The pseudocode of this step is described as follows:

---

**Input:** Relational tables  $X_1, X_2, \dots, X_r$   
**Output:** All the possible conditional faulty sets  $X$

---

```

1 for  $i$  from 1 to  $r$ 
2   IF there exists common fields between  $X$  and  $X_i$ 
3     Then  $X = X \bowtie X_i$ 
4     Else  $X = X \times X_i$ 
5   end if
6 end for
7 return to Step 4
```

---

**Step 4:** Calculate the sum of the two adjacent vertices of each undirected test edge under different conditional faulty sets and  $D(F_i, F_j)$ .

The pseudocode of this step is given below.

---

**Input:** All the possible conditional faulty sets  $X$   
**Output:** The sum of the two adjacent vertices of each undirected test edge under different conditional faulty sets and  $D(F_i, F_j), 1 \leq i < j \leq m$

---

```

1 for each edge  $(u, v) \in E(G)$ 
2   for each conditional fault set  $F_i$ 
3     Calculate  $(u + v) |_{F_x}$ 
4   end for
5 end for
6 for  $i$  from 1 to  $m$  ( $m$  represents the total number of conditional faulty sets)
7   for  $j$  from  $i + 1$  to  $m$ 
8     Calculate  $D(F_i, F_j)$ 
9   end for
10 end for
11 return to Step 5
```

---

**Step 5:** Call the decision function  $J(F_1, F_2, \dots, F_p)$  to determine whether the system is conditionally t-diagnosable or not and derive  $t_c(G)$ .

Let all those conditional faulty sets which have less than  $i$  faulty vertices be  $F_1, F_2, \dots, F_p$ .  $J(F_1, F_2, \dots, F_p) = 0$  represents the system is conditionally t-diagnosable, with  $t = i$ .  $t_c(G)$  is the maximum value of  $t$ .

Step 5 can be described by the following pseudocode.

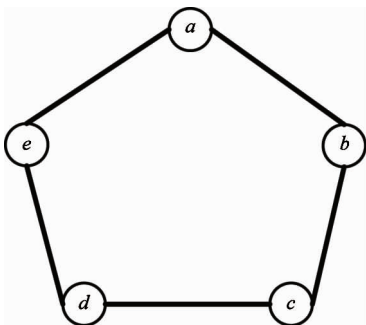
**Input:**  $D(F_i, F_j), 1 \leq i < j \leq m$

**Output:**  $t_c(G)$

- 1 for  $i$  from 1 to the max number of faulty vertices of all the faulty sets in  $X$
- 2 Calculate  $J(F_1, F_2, \dots, F_p)$ , where  $F_1, F_2, \dots, F_p$  are all the conditional faulty sets which have less than  $i$  faulty vertices.
- 3 if  $J(F_1, F_2, \dots, F_p) = 0$
- 4 then the system is conditionally  $t$ -diagnosable, with  $t = i$
- 5 else the system is not conditionally  $t$ -diagnosable
- 6 end if
- 7 end for
- 8  $t_c(G) =$  the maximum value of  $t$  such the system is conditionally  $t$ -diagnosable

Illustrated by the example of Fig. 1, conditional faulty set equations can be constructed as Eq. (1) then to obtain all the relational tables as shown in Table 4. Finally, the new relational table  $X$  can be got by  $X = X_1 \times X_2 \bowtie X_3 \bowtie X_4 \bowtie X_5$ . The result of  $X$  is shown in Table 5.

As shown, there are 11 conditional faulty sets, where  $F_1$  has no faulty vertex, each conditional faulty set of  $\{F_2, F_3, \dots, F_6\}$  has only one faulty vertex, and each conditional faulty set of  $\{F_7, F_8, \dots, F_{11}\}$  has two faulty vertices. The maximum number of faulty vertices of all the possible conditional faulty sets is 2. That is to say,  $t_c(G) \leq 2$ . The sums of the two adjacent vertices of each undirected test edge under different conditional faulty sets are shown in Table 6. And  $D(F_i, F_j) = 0$  for  $1 \leq i < j \leq 11$  as shown in Table 7. All the possible conditional faulty sets in which the number of faulty vertices does not exceed 1 are  $F_1, F_2, \dots, F_6$ .  $J(F_1, F_2, \dots, F_6) = 0$  means the system is conditionally  $t$ -diagnosable, with  $t = 1$ . Next, all the possible conditional faulty sets in which the number of faulty vertices does not exceed 2 are  $F_1, F_2, \dots, F_{11}$ . also  $J(F_1, F_2, \dots, F_{11}) = 0$  implies that the system is conditionally  $t$ -diagnosable, with  $t = 2$ . By  $t_c(G) \leq 2$ ,  $t_c(G) = 2$  is got.



**Fig. 1** A system consisting of 5 vertices

$$\begin{cases} ac = 0 \\ bd = 0 \\ ce = 0 \\ ad = 0 \\ be = 0 \end{cases} \quad (1)$$

Table 4 Relational tables corresponding to Eq. (1)

Equation	Relational table name	Relational table	
$ac = 0$	$X_1$	a	c
		0	0
		1	0
		0	1
$bd = 0$	$X_2$	b	d
		0	0
		1	0
		0	1
$ce = 0$	$X_3$	c	e
		0	0
		1	0
		0	1
$ad = 0$	$X_4$	a	d
		0	0
		1	0
		0	1
$be = 0$	$X_5$	b	e
		0	0
		1	0
		0	1

### 4 Conclusion

Conditional diagnosability is a new measure of diagnosability which claims that each vertex has at least one fault free neighbor. Therefore, all the fault processors can be identified if the number of fault processors in a system is less than the conditional diagnosability and any faulty set cannot contain all neighbors of any processor. As a result a conditional diagnosability algorithm is more important, which can determine conditional diagnosability of any system. With the continuous development of large-scale integration, multiprocessor systems may have hundreds of processors, especially in supercomputer systems, high-performance parallel computing systems and grid systems, which are

Table 5 All the conditional faulty sets in  $X$

No.	a	b	c	d	e
$F_1$	0	0	0	0	0
$F_2$	0	0	0	0	1
$F_3$	0	0	0	1	0
$F_4$	1	0	0	0	0
$F_5$	0	1	0	0	0
$F_6$	0	0	1	0	0
$F_7$	0	0	1	1	0
$F_8$	0	1	1	0	0
$F_9$	0	0	0	1	1
$F_{10}$	1	0	0	0	1
$F_{11}$	1	1	0	0	0

Table 6 The sums of the two incident vertices

No.	$(a+b)  _{F_x}$	$(b+c)  _{F_x}$	$(c+d)  _{F_x}$	$(d+e)  _{F_x}$	$(e+a)  _{F_x}$
$F_1$	0	0	0	0	0
$F_2$	0	0	0	1	1
$F_3$	0	0	1	1	0
$F_4$	1	0	0	0	1
$F_5$	1	1	0	0	0
$F_6$	0	1	1	0	0
$F_7$	0	1	2	1	0
$F_8$	1	2	1	0	0
$F_9$	0	0	1	2	1
$F_{10}$	1	0	0	1	2
$F_{11}$	2	1	0	0	1

Table 7 The results of  $D(F_i, F_j)$ , for  $1 \leq i < j \leq 11$

$D(F_1, F_2) = 0$	$D(F_2, F_3) = 0$	$D(F_3, F_4) = 0$	$D(F_4, F_5) = 0$	$D(F_5, F_6) = 0$	$D(F_6, F_7) = 0$	$D(F_7, F_8) = 0$	$D(F_8, F_9) = 0$	$D(F_9, F_{10}) = 0$	$D(F_{10}, F_{11}) = 0$
$D(F_1, F_3) = 0$	$D(F_2, F_4) = 0$	$D(F_3, F_5) = 0$	$D(F_4, F_6) = 0$	$D(F_5, F_7) = 0$	$D(F_6, F_8) = 0$	$D(F_7, F_9) = 0$	$D(F_8, F_{10}) = 0$	$D(F_9, F_{11}) = 0$	
$D(F_1, F_4) = 0$	$D(F_2, F_5) = 0$	$D(F_3, F_6) = 0$	$D(F_4, F_7) = 0$	$D(F_5, F_8) = 0$	$D(F_6, F_9) = 0$	$D(F_7, F_{10}) = 0$	$D(F_8, F_{11}) = 0$		
$D(F_1, F_5) = 0$	$D(F_2, F_6) = 0$	$D(F_3, F_7) = 0$	$D(F_4, F_8) = 0$	$D(F_5, F_9) = 0$	$D(F_6, F_{10}) = 0$	$D(F_7, F_{11}) = 0$			
$D(F_1, F_6) = 0$	$D(F_2, F_7) = 0$	$D(F_3, F_8) = 0$	$D(F_4, F_9) = 0$	$D(F_5, F_{10}) = 0$	$D(F_6, F_{11}) = 0$				
$D(F_1, F_7) = 0$	$D(F_2, F_8) = 0$	$D(F_3, F_9) = 0$	$D(F_4, F_{10}) = 0$	$D(F_5, F_{11}) = 0$					
$D(F_1, F_8) = 0$	$D(F_2, F_9) = 0$	$D(F_3, F_{10}) = 0$	$D(F_4, F_{11}) = 0$						
$D(F_1, F_9) = 0$	$D(F_2, F_{10}) = 0$	$D(F_3, F_{11}) = 0$							
$D(F_1, F_{10}) = 0$	$D(F_2, F_{11}) = 0$								
$D(F_1, F_{11}) = 0$									

usually based on an underlying bus structure, or a kind of interconnection networks. However, the high complexity of these systems may threaten their reliability. Hence, an efficient conditional diagnosability algorithm has important theoretical significance and application value, which can be used to evaluate the reliability of multiprocessor systems.

In this paper, the distinguishable measure of pairs of distinct faulty sets have be investigated. By theoretical deduction, an effective decision function  $J(F_1, F_2, \dots, F_m)$  and a novel conditional diagnosability algorithm are presented successfully which can identify whether the system is conditionally t-diagnosable or not directly and obtain  $t_c(G)$  conveniently under the PMC model.

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