Study on intercarrier interference in mobile MIMO-OFDM based on the geometrical one-ring model

Li Juhu(李巨虎)②, Zhi Xiyun, He Zhiqiang, Wu Weiling
(Key Laboratory of Universal Wireless Communications, Ministry of Education, Beijing University of Posts and Telecommunications, Beijing 100876, P.R.China)

Abstract

This paper investigates the distribution of intercarrier interference (ICI) in multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems based on the geometrical one-ring model. Using the spatial and temporal correlation of a geometrical one-ring model, a close-formed expression of intercarrier interference due to the Doppler effect caused by the movement of receiver is derived under the isotropic scattering conditions and non-isotropic scattering conditions. The analytical results are verified by Monte Carlo simulations. We use the generated channels to investigate MIMO-OFDM intercarrier interference under various channel parameters. It can be shown that more than 95% of ICI power comes from five neighboring subcarriers.

Keywords: intercarrier interference (ICI), one-ring model, spatial and temporal correlation, isotropic scattering, non-isotropic scattering

0 Introduction

In recent years, a great deal of attention concentrates on MIMO-OFDM due to its potential of achieving high data rates. Most studies concerning MIMO-OFDM are focused on slowly fading channels. However, in order to support a full mobility in macro cellular environments, the time variation of a fading channel over an MIMO-OFDM block must be considered since it causes a loss of subcarriers orthogonally, leading to intercarrier interference (ICI). The results in ICI and a significant performance loss become more severe as the normalized Doppler frequency increases.

For the macrocells, the scatterers are assumed to be located uniformly inside a circular ring around a mobile station (MS). This model can be reduced to the well-known geometrical one-ring model proposed by Liberti and Rappaport. The geometrical one-ring model[1] has widely been accepted as proper MIMO channel models to design, optimize, and test future mobile systems. Hence, it is highly necessary and useful to investigate the ICI under realistic propagation conditions emulated by one-ring geometrical MIMO channel models.

Several methods have been proposed to reduce the effect of the ICI. Ref.[2] examines the effects of ICI and proposes a method by modeling ICI as an additive Gaussian random process. Ref.[3] studies the performance of ICI on the isotropic scattering condition. Ref.[4] studies the performance of ICI by modeling the time-variant channels within an OFDM block as a $D$th order polynomial function. However, this model is not sufficiently accurate since the Doppler shift depends on the angle of motions at the receiver. From the analysis of the ICI channel matrix[5], it revealed that the interference can be effectively bounded in finite double-side subcarriers centering the desired subcarrier. But the authors do not study the impact of the angle of arrival at the receiver on the ICI of MIMO-OFDM systems.

This paper will focus on the ICI of MIMO-OFDM systems using the wideband MIMO one-ring channel models. We will first discuss the impact of normalized Doppler frequency on the ICI of MIMO-OFDM systems under isotropic scattering conditions and non-isotropic scattering conditions. Secondly we discuss the impact of the angle of motion on the ICI of MIMO-OFDM systems under non-isotropic scattering conditions. The paper is structured as follows. In Section 1, we describe the MIMO-OFDM system model with ICI. In Section 2 we briefly review the wideband MIMO-OFDM channel reference models derived from the geometric one-ring scattering model. Section 3 studies the symbol energy leakage and the ICI due to Doppler spread. Simulations are provided in section 4, and section 5 summarizes our conclusions.

1 System model

① Supported by the National Basic Research Program of China (NO.2009CB320401), the National Key Scientific and Technological Project of China (No.2012ZX03004005-002, No.2010ZX03003-003-01), New Generation Broadband Wireless Mobile Communication Network of Major Special Projects (No.2010ZX03003-001), the Fundamental Research Funds for the Central Universities(2010PTB-03-04)

② To whom correspondence should be addressed. E-mail: ljuhu78@163.com

Received on Apr. 27, 2010
This section will introduce the $N_r$-sub carriers MIMO-OFDM system with $N_r$ transmit antennas and $N_s$ receive antennas. The transmitted signal of $v$ th transmit antenna over a block, including the cyclic prefix, can be given by

$$x_v(t) = T_s \sum_{k=0}^{N-1} X_v[k] e^{j2\pi k t/T_s} - T_p \leq t < T_s$$

where $X_v[k]$ is a modulated symbol on the $k$ th subcarriers sent by the $v$ th transmit antenna. $T_S$ is symbol duration, and $T_P$ is the length of the cyclic prefix. The input data vector can then be written as $X_v = [X_v[0], ..., X_v[N_c-1]]^T$. Assuming the multi-path fading channel contains $L$ resolvable paths, the complex gain between the $v$ th transmit antennas and the $u$ th receive antennas $h_{uv}(t, \tau)$ can be written as

$$h_{uv}(t, \tau) = \sum_{l=1}^{L} \alpha_l g_{uv}(l) \delta(\tau - \tau_l)$$

where $\alpha_l$ represents the delay coefficient of the $l$ th propagation path and $\tau_l$ denotes the corresponding propagation delay. We impose the boundary condition $\sum_{l=1}^{L} \alpha_l^2 = 1$ on the delay coefficient to normalize the mean power of $h_{uv}(t, \tau)$ to unity. For $v = 1, ..., N_r$ and $u = 1, ..., N_s$, $g_{uv}(l)$ denotes the channel gain of the $l$ th propagation path. The time-domain receive signal of $u$ th receive antennas can be written as

$$y_u(t) = \sum_{v=1}^{N_r} \sum_{l=1}^{L} h_{uv}(t, \tau) x_v(t-\tau) d\tau + w_u(t)$$

where $w_u(t)$ is an additive white Gaussian noise (AWGN). After discarding the cyclic prefix, the demodulated signal is obtained by performing inverse Fourier transform to $y_u(t)$ as follows:

$$Y_u[m] = T_s^{-1/2} \int_{-T_s/2}^{T_s/2} y_u(t) e^{-j2\pi km} dt$$

$$= T_s^{-1/2} \sum_{l=1}^{L} \sum_{v=1}^{N_r} \alpha_l X_v[k] e^{j2\pi km/T_s} \int_{-T_s/2}^{T_s/2} g_{uv}(l) e^{-j2\pi km} dt + W_u[m]$$

where $X_v[k] = [X_v[k], X_v[k], ..., X_v[k]]^T$ leaking to the $u$ th receive antenna of $m$ th subcarriers is an additive white Gaussian noise (AWGN).

### 2 MIMO-OFDM channel models

#### 2.1 The geometrical one-ring model

The geometrical one-ring MIMO-OFDM channel model is shown in Fig.1. It is assumed that there are $N$ local scatter $S_v^n(n = 1, 2, ..., N)$ located on a ring around the receiver.\(^1\)

![Fig.1 One-ring model for an $N_r \times N_s$ MIMO-OFDM channel](image)

We will partition all scatters located on the ring around the receiver into $L$ cluster pairs. The number of scatter within the $l$ th cluster pairs, denoted by $N_l$, must fulfill the boundary condition $\sum_{l=1}^{L} N_l = N$. We use a set $I_l$ to denote the scatter location in the cluster pairs. The $n$ th scatter $S_v^n$ is located in the $l$ th cluster pairs if $n \in I_l$. It was shown that the time-variant complex channel gain $h_{uv}(t)$, which describes the link from the $v$ th transmit antenna to the $u$ th receive antenna, can be expressed as\(^1\)

$$h_{uv}(t) = \sum_{l=1}^{L} \alpha_l g_{uv}(l) \delta(\tau - \tau_l)$$

For $v = 1, ..., N_r$ and $u = 1, ..., N_s$, $g_{uv}(l)$ denotes the channel gain of the $l$ th propagation path, which can be expressed as

$$g_{uv}(l) = \lim_{h \to \infty} \frac{1}{N_0} \sum_{n \in I_l} \alpha_{n,v,l} b_{n,u,l} e^{(j2\pi f_0 l)\theta_{n,v,l}}$$

where

$$\alpha_{n,v,l} = e^{j(\pi/2 - 2\pi v + \pi l) r_{n,v,l} \sin(\theta_{n,v,l})}$$

$$b_{n,u,l} = e^{j(\pi/2 - 2\pi u + \pi l) \cos(\psi_{n,u,l} - \delta_{q,l})}$$

and $\lambda$ describes the carrier’s wavelength and $f_{max}$ denotes the maximum Doppler frequency. The phases $\theta_{n,v,l}$ are independent and identically distributed random variables, which are uniformly distributed over $[0, 2\pi]$. The antenna spacing at the transmitter and the receiver are denoted by $\delta_{r}$ and $\delta_{s}$ respectively, and the multi-element antenna tilt angles...
are described by $\alpha_r$ and $\alpha_k$. The angle of motion denoted by $\alpha_r$ and $\phi_{r}^{\text{max}}$ describes the maximum angle of departure seen at the transmitter. The angle of arrival (AOA) is described by $\phi_{k}^{(\alpha)}$. In the reference model, the AOA is an independent random variable determined by the distribution of the local scatter. The correlation between $h_{v}(t, \tau)$ and $h_{v'}(t, \tau')$ is defined as

$$r_{w,v',v}(t_1-t_2) = E[h_{v}(t, \tau)h_{v'}(t, \tau')]$$

(8)

Since $h_{v}(t, \tau)$, $\ell \in [1, L]$ is uncorrelated, $r_{w,v',v}(t_1-t_2)$ can be written as

$$r_{w,v',v}(t_1-t_2) = \sum_{\ell} \alpha_r^2 r_{w,v',v}(t_1-t_2)$$

(9)

where $r_{w,v',v}(t_1-t_2)$ describing the spatial and temporal correlation function can be defined as

$$r_{w,v',v}(t_1-t_2) = E[g_{w,v}(t_1)g_{v',v}(t_2)] = \int_{-\pi}^{\pi} e^{-j2\pi f_{\text{max}} \cos(\phi_{k} - \phi_{k'})} p_{r}(\phi_{k}) d\phi_{k}$$

(10)

Under $u'=u, v'=v$, the preceding equation can be simplified as

$$r_{w,v',v}(t_1-t_2) = \int_{-\pi}^{\pi} e^{-j2\pi f_{\text{max}} \cos(\phi_{k} - \phi_{k'})} p_{r}(\phi_{k}) d\phi_{k}$$

(11)

which describes the temporal autocorrelation function of the $\ell$ th propagation path. So the temporal autocorrelation function can be given by

$$r_{w,v}(t_1-t_2) = \sum_{\ell} \alpha_r^2 r_{w,v}(t_1-t_2) = \sum_{\ell} \alpha_r^2 \int_{-\pi}^{\pi} e^{-j2\pi f_{\text{max}} \cos(\phi_{k} - \phi_{k'})} p_{r}(\phi_{k}) d\phi_{k}$$

(12)

### 2.2 Distribution of the AOA

Many different scatters distributions have been proposed to characterize the distribution of the AOA $\phi_k$ in the literature, such as uniform, Gaussian, wrapped Gaussian, and the cardiods probability density function. In this paper, the von Mises probability density function (PDF) is used, which approximates many of these distribution and provides mathematical convenience leading to closed-form solutions for many problems. The von Mises PDF is defined as

$$p_{r}(\phi_{k}) = \frac{1}{2\pi I_0(\kappa)} \exp[\kappa \cos(\phi_{k} - \mu_{\ell})], \quad \phi_{k} \in [-\pi, \pi)$$

(13)

here $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero, $\mu_{\ell} \in [-\pi, \pi)$ is the mean angle at which the scatters are distributed on the ring, and $\kappa$ controls the spread of scatters around this mean. $\kappa = 0, p_{r}(\phi_{k}) = 1/2\pi$ blows the isotropic scattering. As $\kappa$ increases, the scatters become more clustered around $\mu_{\ell}$ and the scattering becomes increasingly non-isotropic.

### 3 Symbol energy distribution

Since $g_{w,v}(t; \ell \in [1, L])$ are uncorrelated, the energy of $\mathbf{X}[k] = [X_1[k], \ldots, X_N[k]]^T$ leaking to the $u$ th receive antennas of $m$ th subcarrier can be written as

$$\Phi_{u,m,k} = E[X_u[k]X_\ell^*[k]] = \frac{1}{T_e} \sum_{\ell \in \ell_u \ell_v \ell_N} E[X_u[k]X_\ell^*[k]]$$

(14)

$$\times \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \sum_{\ell=1}^{\ell_{N}} \alpha_r^2 r_{w,v}(t_1-t_2) e^{-j2\pi(\mu_{\ell} - \phi_{k})} dt_1dt_2$$

Since the symbol of $k$ th subcarrier is uncorrelated between $v$ th transmit antenna and $v'$ th transmit antenna, it can be expressed as

$$E[X_u[k]X_\ell^*[k]] = \begin{cases} \epsilon_u & v = v' \forall k \\ 0 & v \neq v' \forall k \end{cases}$$

(15)

Using Eq.(12) and Eq.(15), Eq.(14) can be simplified as

$$\Phi_{u,m,k} = \epsilon_u \sum_{\ell \in \ell_u \ell_v \ell_N} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \alpha_r^2 r_{w,v}(t_1-t_2) e^{-j2\pi(\mu_{\ell})} dt_1dt_2$$

(16)

where $\mathbf{q} = m - k$, energy of $\mathbf{X}[k] = [X_1[k], \ldots, X_N[k]]^T$ distributed to subcarriers $k - Q$ to $k + Q$ can be expressed as

$$\Psi_{u,Q} = \sum_{q=-Q}^{Q} \Phi_{u,q}$$

(17)

Using Ref.[9], Eq.(17) can be simplified as

$$\Psi_{u,Q} = \epsilon_u \sum_{\ell \in \ell_u \ell_v \ell_N} \int_{-\pi}^{\pi} r_{w,v}(T_x)(1-|x|) \left[ \sum_{q=-Q}^{Q} e^{-j2\pi q} \right] dx$$

(18)

From Ref.[10], the total ICI power is

$$P_{u,CIJ} = \sum_{q=-Q}^{Q} \Phi_{u,q} = \epsilon_u \sum_{\ell \in \ell_u \ell_v \ell_N} \int_{-\pi}^{\pi} r_{w,v}(T_x)(1-|x|) \left[ \sum_{q=-Q}^{Q} e^{-j2\pi q} \right] dx$$

(19)

If the distribution of AOA is the von Mises probability density function (PDF), the temporal autocorrelation function Eq.(12) can be given by

$$r_{w,v}(T_x) = \sum_{\ell=1}^{\ell_{N}} \alpha_r^2 I_0(\kappa)$$

(20)
where \( \beta_i = \sqrt{\kappa_i^2 - 4\pi^2 f_{\text{max}}^2 (T_i x)^2 - j4\pi \kappa_i f_{\text{max}} T_i x \cos(\alpha_f - \mu_i)} \)
and \( I_0(\cdot) \) is the modified Bessel function of the first kind of order zero. Plug Eq.(20) into Eq.(18), we have
\[
\psi_{\alpha,0} = N_r \alpha \int \left( \sum_{i=1}^{\infty} \frac{\alpha_i^2 I_0(\beta_i)}{I_0(\kappa_i)} \right) \left( 1 - |x| \right) \sin \left( (Q + \frac{1}{2}) 2\pi x \right) \sin \pi x \, dx \quad (21)
\]
Plug Eq.(20) into Eq.(19), the total ICI power is
\[
P_{\alpha,\text{ICI}} = N_r \alpha \int \left[ 1 - \sum_{i=1}^{\infty} \alpha_i^2 \frac{I_0(\beta_i)}{I_0(\kappa_i)} \right] \left( 1 - |x| \right) \sin \pi x \, dx \quad (22)
\]
ICI can be well quantified by using the carrier-to-interference ratio \([3]\). In order to quantify the combined effects of both normalized Doppler shift \( f_{\text{max}} T_i x \) and the motion of angle \( \alpha_f \), we derive carrier to interference ratio as
\[
\frac{P_{\text{carrier}}}{P_{\text{interference}}} = \frac{\psi_{\alpha,0} = N_r \alpha \int \left( \sum_{i=1}^{\infty} \frac{\alpha_i^2 I_0(\beta_i)}{I_0(\kappa_i)} \right) \left( 1 - |x| \right) \sin \left( (Q + \frac{1}{2}) 2\pi x \right) \sin \pi x \, dx}{\psi_{\alpha,\text{ICI}}} = N_r \alpha \int \left[ 1 - \sum_{i=1}^{\infty} \alpha_i^2 \frac{I_0(\beta_i)}{I_0(\kappa_i)} \right] \left( 1 - |x| \right) \sin \pi x \, dx \quad (23)
\]
which Eq.(21), Eq.(22) and Eq. (23) describe the distribution of intercarrier interference on non-isotropic scattering condition. Under \( \kappa = 0, p(\phi_\kappa) = 1/2\pi \), so Eq.(21), Eq.(22) and Eq.(23) can be simplified as
\[
\psi_{\alpha,0} = N_r \alpha \int \left( \sum_{i=1}^{\infty} \frac{\alpha_i^2 I_0(\beta_i)}{I_0(\kappa_i)} \right) \left( 1 - |x| \right) \sin \left( (Q + \frac{1}{2}) 2\pi x \right) \sin \pi x \, dx \quad (24)
\]
\[
P_{\alpha,\text{ICI}} = N_r \alpha \int \left[ 1 - \sum_{i=1}^{\infty} \frac{I_0(\beta_i)}{I_0(\kappa_i)} \right] \left( 1 - |x| \right) \sin \pi x \, dx \quad (25)
\]
\[
\frac{P_{\text{carrier}}}{P_{\text{interference}}} = \frac{\psi_{\alpha,0} = N_r \alpha \int \left( \sum_{i=1}^{\infty} \frac{\alpha_i^2 I_0(\beta_i)}{I_0(\kappa_i)} \right) \left( 1 - |x| \right) \sin \left( (Q + \frac{1}{2}) 2\pi x \right) \sin \pi x \, dx}{\psi_{\alpha,\text{ICI}}} = N_r \alpha \int \left[ 1 - \sum_{i=1}^{\infty} \frac{I_0(\beta_i)}{I_0(\kappa_i)} \right] \left( 1 - |x| \right) \sin \pi x \, dx \quad (26)
\]
where \( J_0(\cdot) \) is the zeroth order Bessel function of the first kind. \( r_{\alpha,0}(T_i x) = J_0(2\pi f_{\text{max}} T_i x) \) are the well-known characterizing isotropic scattering channels in Ref.[11]. Eq.(24), Eq. (25) and Eq.(26) describe the distribution of intercarrier interference on isotropic scattering condition. Eq.(26) has been denoted in Ref.[13].

### 4 Simulation results

In the section, the effect of spatial and temporal correlation on the power of ICI is investigated. We can apply the concept of deterministic channel modeling described in Ref.[12]. First we precede the infinite number of scatters by a finite value \( N \). Secondly we determine the distribution of AOA and the phases \( \theta_{\alpha,i} \). We assume that the distribution of the \( \ell \) th cluster follows the von Mises distribution, where the parameters \( \mu_i \) and \( \kappa_i \) are determined as described in Ref.[12].

The phases \( \theta_{\alpha,i} \) can be generated from a random generator with a uniform distribution over \([0, 2\pi]\). As a result, the channel impulse response can be obtained by using Eq.(6) and Eq.(7). The expressions of Eq.(21), Eq.(22), Eq.(23), Eq.(24), Eq.(25) and Eq.(26) can be applied to any array configuration.

Here, uniform linear arrays (ULAs) are considered where the antenna spacing at the transmitter and receiver is given by \( \delta_x = \delta_y = \lambda \), respectively. The angle of transmit and receive antenna arrays is given by \( \alpha_x = \alpha_y = 90^\circ \), respectively. The angle of motion is given by \( \alpha_f \). The carries frequency is set to 3.4 GHz. the number of transmit and receive antennas is set to 2, i.e. \( N_r = N_h = 2 \)

![Fig.2](image)

**Fig.2** Under \( \kappa = 0, p(\phi_\kappa) = 1/2\pi \), normalized symbol energy distribution with isotropic scattering conditions

Under \( \kappa = 0, p(\phi_\kappa) = 1/2\pi \), AOA is a uniform distribution, the exact normalized symbol energy distribution \( \psi_{\alpha,0}/N_r \varepsilon_s \) are shown in Fig.2 for different normalized Doppler frequencies \( f_s T_i \). When \( f_s T_i = 0.1 \), more than 95% of \( X[k] \) energy is distributed on the \( k \) th subcarrier and its one neighboring subcarriers. If Doppler frequency increases, more symbol energy leaks to neighboring subcarriers. When \( f_s T_i = 0.9 \), more than 95% of the symbol energy is spread over three subcarriers. When \( \kappa = 0, p(\phi_\kappa) = 1/2\pi \), the exact normalized ICI power distribution \( P_{\alpha,\text{ICI}}/N_r \varepsilon_s \) is shown in Fig.3 and carrier-to-interference ratio distribution is shown in Fig.4.

The exact normalized symbol energy distributions \( \psi_{\alpha,0}/N_r \varepsilon_s \) with different angles of motion \( \alpha_f \) are shown in Fig.5 under the non-isotropic scattering conditions. When \( \alpha_f = 0 \) or \( \alpha_f = \pi \), the angle of motions parallel with the angle of incident wave, intercarrier interference is the maximum and more energy is distributed on neighboring subcarriers. But the
angle of motions Perpendicular to the angle of incident wave, intercarrier interference is the least and less energy is distributed on neighboring subcarriers.

Fig.3 Under $\kappa = 0$, $p(\phi_k) = 1/2\pi$, total ICI power distribution

Fig.4 Under $\kappa = 0$, $p(\phi_k) = 1/2\pi$ carrier-to-interference ratio distribution with different normalized Doppler frequencies

Fig.5 Normalized symbol energy distribution with various normalized Doppler frequencies under non-isotropic scattering conditions

The exact normalized ICI power distribution $P_{ICIT}/N_0\epsilon_s$ is shown in Fig.6 under non-isotropic scattering conditions. The carrier-to-interference ratio comparison with different motion angle $\alpha_m$ and normalized Doppler frequencies $f_sT_s$ are shown in Fig.7.

5 Conclusion

In this paper, we studied the ICI and energy leakage that is caused by the time-varying mobile channel on isotropic scattering conditions and the non-isotropic scattering conditions. Under the non-isotropic scattering conditions, the influences of the angles of motion and normalized Doppler frequency have been discussed separately. We show that more than 95% of ICI power comes from five neighboring subcarriers with normalized Doppler frequency. These results motivate our developments of low-complexity MMSE and DFE receivers for ICI suppression.
References


Li Juhu, born in 1978. Now he is a currently working towards the Ph.D. degree in Beijing University of Posts and Telecommunications. He received his B.S. and M.S. degrees from Lanzhou University in 2000 and 2003 respectively. His research interests include cooperative communications and MIMO communication.